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Qualitative Comparative Analysis: A Discussion of Interpretations

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The article discusses interpretations of 'Qualitative Comparative Analysis' (QCA) proposed by Charles Ragin. The first section argues that QCA can be understood alternatively as a method of data description or as a method for the construction of deterministic functional models. It is shown that thinking in terms of models is required for generalizations. The second section discusses causal interpretations of such models. It is argued that one can use deterministic models without supposing a deterministic metaphysics. The third section briefly introduces stochastic functional models and shows how they can be used for QCA applications. In addition to showing that QCA can be well understood as a specific method of model construction, the article argues that deterministic and stochastic functional models are quite similar and, depending on the application context and the available data, both kinds of models could be useful.

Introduction

Qualitative comparative analysis (QCA) was first proposed by Charles Ragin in 1987 as a method for analyzing data sets consisting of binary variables (Ragin, 1987). The basic idea was to represent such data by Boolean functions. In the meantime, Ragin (2000, 2008) has extended the method to allow constructions of fuzzy set relations; further extensions allow dealing with variables having more than two values.[1]

The methods of QCA have been used in many applications, in particular when researchers deal with a small or medium number of identifiable cases.[2] In discussions of these methods, it has been proposed to distinguish between understanding QCA as a 'research approach' and as a 'data analysis technique' (Wagemann and Schneider, 2010). As a research approach, QCA tries to combine qualitative and quantitative research methods (see also Ragin, 2008). The present article focuses on QCA as a data analysis technique. It is argued that the methods proposed under the heading of QCA can be interpreted in different ways, and that the differences are important for the question whether, and how, QCA can be used for theoretical, in particular for causal, inferences.

The first section shows that the basic form of QCA can be understood alternatively as a method of data description or as a method for the construction of deterministic functional models. It is shown that thinking in terms of models is required for generalizations. The second section discusses causal interpretations of such models. It is argued that one can use deterministic models without supposing a deterministic metaphysics. The third section briefly introduces stochastic functional models and shows how they can be used for QCA applications. In addition to showing that QCA can well be understood as a specific method of model construction, the article argues that deterministic and stochastic functional models are quite similar and, depending on the application context and the available data, both kinds of models could be useful. The article ends with brief conclusions.

Understanding the QCA Approach

In this section, I discuss the basic form of QCA that consists in the construction of Boolean functions based on binary variables. I show that the technique can be used both for descriptive and analytical purposes. In order to understand analytical applications, aiming at causal inferences, I consider QCA as a method of model construction. This also allows to compare QCA with stochastic forms of model construction that are prevalent in statistical social research.

An Introductory Example

I begin with an artificial example that was used by Ragin (1987, pp. 95–101) to explain his approach. The data are presented as a 'hypothetical truth table showing three causes of successful strikes'. There are four binary variables:

$S = 1$ if the strike is successful, $S = 0$ otherwise

$A = 1$ if booming product market, $A = 0$ otherwise

$B = 1$ if threat of sympathy strikes, $B = 0$ otherwise

$C = 1$ if large strike fund, $C = 0$ otherwise

The data are assumed to be given as shown in Table 1; the last column provides the number of cases.

S is considered as dependent, *A*, *B*, and *C* are taken as independent variables. As the table shows, there are observations for each of the eight possible configurations of (= assignments of values to) the independent variables and, furthermore, in all cases belonging to the same configuration of the independent variables, the dependent variable has the same value, either 0 or 1.

Table 1.

A	*B*	*C*	*S*	Cases
1	0	1	1	6
0	1	0	1	5
1	1	0	1	2
1	1	1	1	3
1	0	0	0	9
0	0	1	0	6
0	1	1	0	3
0	0	0	0	4

(How these features of the data can be relaxed will be discussed below.)

The method proposed by Ragin aims at the construction of a function that shows how values of *S* depend on configurations (values) of the independent variables. It is basically a 'technique of data reduction that uses Boolean algebra to simplify complex data structures in a logical and holistic manner' (Ragin, 1987, viii). In the present example the method produces the function

$$S = A.C + B.C' \qquad (1)$$

to read: $S=1$ iff $A=1$ and $C=1$ or $B=1$ and $C=0$.[3] The function obviously shows how, in this example, *S* depends on *A*, *B*, and *C*. Ragin also claims that functions constructed in this way can be given a causal interpretation:

> The final, reduced equation [$S = A.C+B.C'$] shows the two (logically minimal) combinations of conditions that cause successful strikes and thus provides an explicit statement of multiple conjunctural causation (Ragin, 1987, p.98).

The implied understanding of causality will be discussed in the 'Causal Interpretations' section. A preliminary question concerns in which sense QCA is not just a technique of data reduction but also a method of constructing models which, in some sense, transcend the actual data. This will be discussed in the remainder of the present section.

Statistical Data and Variables

In order to approach the question one first needs a distinction between variables that represent data and variables to be used in the formulation of models. Variables that serve to represent data, that is, information about realized facts, will be called *statistical variables*. Such variables can formally be defined as functions having the form $X : \Omega \rightarrow \mathcal{X}$. Ω, the variable's domain (also called its *reference set*), comprises a set of cases to which the data relate, and $\mathcal{X}$ denotes a property space.[4] Then, for each element $\omega \in \Omega$, the value of the statistical variable, $X(\omega) \in \mathcal{X}$, characterizes the case represented by ω. It will be assumed that the property space has a numerical representation and can therefore be considered as some subset $\mathcal{X} \subseteq \mathbf{R}$.[5]

The example of the subsection 'An Introductory Example' can serve to illustrate the notion. In this example, the reference set Ω has 38 elements corresponding to the 38 cases for which data are available.

The statistical variable consists of four components and may be written as

$$(A, B, C, S) : \Omega \rightarrow \{0, 1\}^4$$

The important point is that statistical variables represent data which refer to realized facts. In this example, the variable (A, B, C, S) refers to a set of 38 strikes which are assumed to have actually taken place (somewhere at some time) and have actually exhibited the features indicated by the variable's values. Using statistical variables as a conceptual framework, therefore, only allows descriptive statements about observed or hypothetically assumed facts.

Descriptive and Modal Generalizations

So the question arises how to understand statements which, by transcending the given data, are in some sense more general. There are at least two quite different ideas. One idea takes the cases for which data are actually available to be a subset of a larger population of cases and intends a generalization to that population. The conceptual framework of statistical variables allows an explicit formulation. Suppose a statistical variable, say $X : \Omega \rightarrow \mathcal{X}$, represents the available data.[6] This allows statistical statements about the reference set Ω, basically provided by the distribution of X which will be denoted by P[X].[7] Ω is then considered as a subset of a larger population Ω^* for which an analogously defined variable, say $X^* : \Omega^* \rightarrow \mathcal{X}$, can be assumed. The goal of the generalization is a statement about $\mathrm{P}[X^*]$, the distribution of X^* in the population Ω^*, or some quantity derived from that distribution. For example, under certain circumstances it might be reasonable to believe that $\mathrm{P}[X^*] \approx \mathrm{P}[X]$. In any case, the result is a descriptive statement about the distribution of X^* in the population for which the generalization is desired; the approach will therefore be called *descriptive generalization*.

While often reasonable, a descriptive approach to generalization has, in fact, severe limitations. The most important limitation results from the notion of a population. In order to be used as a reference set for a statistical variable, the elements of a population can only represent cases which actually do exist or have existed in the past. Moreover, in order to think of Ω as a random sample from Ω^*, all elements of the population must exist at the time of generating the sample. For example, it might well be possible to think of 38 observed strikes as being a subset of a larger population of strikes, say Ω^*; but elements of Ω^* may only refer to strikes that have actually taken place, not to strikes that might take place somewhere in the future. For otherwise the variables have no operational meaning.

However, interest in future possibilities often provides the main reason for an interest in generalizations. One might be interested, for example, in how the possible success of a strike depends on conditions. This question no longer refers to a definite set of realized facts but is a modal question that concerns the dependency of possibilities on conditions.[8] Such questions cannot be answered in the conceptual framework of descriptive generalizations, but require a different kind of generalization that will be called *modal generalization*.

Modal Generalizations with Functional Models

The main linguistic tool for the formulation of modal generalizations are rules. Corresponding to different kinds of modal generalizations, there are different kinds of rules. A rule might say, for example, what, in a situation of a certain kind, might happen, or will probably happen, or should be done, or can be achieved by performing some specified action. Scientific research is often concerned with *causal rules* that are intended to show how facts, or events, of some kind depend on other facts and/or events.[9] Modal generalizations may then be called *causal generalizations*.

A widespread approach to causal generalizations consists in the construction of *functional models*, that is, models which show how one or more endogenous variables depend on one or more exogenous variables.[10] The important point is that these variables, contrary to statistical variables, do not represent realized facts, but are intended to serve modal thinking about dependencies between possible facts and/or events. These variables will therefore be called *modal variables* and a specific notation will be used. They will be marked by a single point if stochastic or by double points if deterministic.

For the moment I only consider *deterministic models* consisting of deterministic variables connected by deterministic functions.[11] In the simplest case there is just one exogenous variable, say $\ddot{X}$, and one endogenous variable, say $\ddot{Y}$, having ranges $\mathcal{X}$ and $\mathcal{Y}$, respectively,[12] and it is assumed that they are connected by a function, say $f : \mathcal{X} \rightarrow \mathcal{Y}$, such that for each value $x \in \mathcal{X}$ there is a unique value $f(x) \in \mathcal{Y}$ for the variable $\ddot{Y}$. The model may then be depicted as $\ddot{X} \rightarrow \ddot{Y}$. The model is called *deterministic* because the variables are connected by a deterministic function, that is, a function which assigns to each configuration of its arguments a *unique* value of the dependent variable.

Of course, it does not suffice just to assume the existence of a function. Knowing a function one should be able to calculate its value for any possible arguments. Therefore, given this knowledge, the function can be taken as a rule which can effectively be used. Correspondingly, the functional model can be viewed as describing one or several (in some way connected) rules.

Following this understanding, to use, or apply, a functional model simply means to use the rule (or rules) formulated by the model for some calculation. This, of course, presupposes a context which provides values for the model's exogenous variables. However, the model is silent about where these values come from. They might result from observation or from hypothetical assumption. The model only describes how its endogenous variables get their values if values of its exogenous variables are given. Correspondingly, the model has no relationship to any concrete situations, and in particular does not describe any facts or events which were realized in those situations. Instead, the model relates *in an unspecified sense* to a generic situation with the only requirement that the situation can generate values of the model's exogenous and endogenous variables.

QCA as a Method of Model Construction

The example of the subsection 'An Introductory Example' can serve to illustrate the construction of a deterministic functional model and the contribution of QCA. There are four steps.

(a) In a first step one defines the model's endogenous and exogenous variables. In this example, there is a single endogenous variable, $\ddot{S}$, and there are three exogenous variables, $\ddot{A}$, $\ddot{B}$, and $\ddot{C}$. All variables have ranges $\{0, 1\}$ with meanings analogously defined to the meanings of the statistical variables introduced in the subsection 'An Introductory Example'. However, $\ddot{A}$, $\ddot{B}$, $\ddot{C}$, and $\ddot{S}$ are now modal variables; they refer to a generic situation in which a strike is assumed to take place and may happen to be successful or not, depending on the circumstances.

(b) In a second step one specifies the functional connections between the exogenous and endogenous variables. In this example, the model may be depicted as $(\ddot{A}, \ddot{B}, \ddot{C}) \rightarrow \ddot{S}$. There is a single deterministic function

$$f : \{0, 1\}^3 \rightarrow \{0, 1\},$$

which provides for each configuration $(a,b,c) \in \{0,1\}^3$ a unique value $s = f(a,b,c)$ of the variable $\ddot{S}$.

(c) In a third step one uses available data in order to find a numerical specification of the function(s) defined in the second step. In this example, the data as tabulated in Table 1 already provide a numerical specification of the function *f*. Knowing this table, one can effectively calculate a value of $f(a,b,c)$ for each configuration of the arguments, and in this sense one then knows the function as a rule.

(d) Finally, one can use Boolean algebra to find a simplified representation of the function(s). In this example, one finds

$$\ddot{S} = \ddot{A}.\ddot{C} + \ddot{B}.\ddot{C}', \qquad (2)$$

which is formally analogous to (1).

Notice, however, that (2) and (1) have different explications. Equation (1) employs statistical variables defined by the data in Table 1 and expresses a descriptive statement about these data. In contrast, Equation (2) characterizes the function of a functional model and is formulated in terms of modal variables which do not refer to any data. While it is true that the equation has been found by using the data in Table 1, it nevertheless does not make a descriptive statement about these data. In fact, Equation (2) does not make a descriptive statement at all; it formulates a rule.

Descriptive Statements and Rules

The distinction between descriptive statements and rules is of fundamental importance. Descriptive statements serve to state facts and can be true or false. Rules, on the other hand, do not state facts and cannot (therefore) be true or false. Instead, rules are best understood as aids to support people in their thinking and doing. This understanding also applies to the functions defined by a functional model. These functions formulate rules which can be used for predictions: given values of the model's exogenous variables, they can be used to predict values of its endogenous variables.

Of course, predictions can be wrong. But given that a prediction turns out to be wrong, this will not falsify the rule that was used for the prediction. Think, for example, of the rule: if the bell-push is pressed, the bell will ring. The rule will not be falsified when the bell does not ring after the bell-push was pressed. Rather, that the bell did not ring might be explained by some circumstances not explicitly taken into account in the formulation of the rule.

This is of general importance: the formulation of a predictive rule cannot explicitly take into account all conditions required for correct predictions. This is true, in particular, of functional models. Since these models refer to generic situations they can take into account at best a few conditions on which their endogenous variables depend. A functional model might, therefore, lead to a wrong prediction when applied in a concrete situation. But again, this will not falsify the model. Functional models, like rules, are not true or false but more or less useful.

When data contradict a functional relationship assumed in a model, this will not falsify the model. Such observations might suggest changes in the formulation of the model. They do not, however, force a reformulation because the model has not the task to describe data. Of course, a model cannot be used to explain the occurrence of data that contradict the model. But even then the model might be useful to define the question of interest, namely, why a contradicting outcome occurred in the given situation. In many cases, an answer will not require a modified model but a thorough investigation of the circumstances which prevailed in the concrete situation.

Why Minimal Function Descriptions?

Viewed as a technique of Boolean function minimization, QCA is just the final step in the construction of a deterministic functional model (based on a Boolean function). This step only changes the presentation, not the essential content of the functional relationship established by the model. So what can be achieved by the minimization procedure? Ragin's main argument is that this procedure can find constellations of causally relevant conditions. Griffin and Ragin (1994, p. 9) gave the following formulation:

> The procedure's explanatory power derives from how it uses Boolean algebra as a data-reduction tool. By comparing combinations of causal factors, QCA trims them of components logically unnecessary for the presence of the outcome in one or more cases. What remains after data reduction – logically irreducible and nonredundant combinations of attributes – is viewed as a set of causal conjunctures that are sufficient for the occurrence of the outcome (Ragin, 1987).

In order to understand the idea, I first briefly consider the minimization procedure. Let $Y = f(X_1, \ldots, X_m)$ denote a Boolean function (all variables are binary). A complete truth-table has 2^m rows; each row will be called a *configuration* (or *constellation*) of the independent variables. These variables can be used to construct *multiplicative expressions* having the form $Z_1.Z_2...Z_k$, where each Z_i is one of the independent variables or its negation, e.g. $X_1.X_2$ or $X_2.X_3.X'_5$. An expression $Z_1.Z_2...Z_k$ will be called an *implicant* if $Z_1.Z_2...Z_k = 1$ implies that $Y = 1$; and it will be called a *minimal* or *prime implicant* if no proper part of it is already an implicant. These definitions finally allow one to formulate the following

> Minimization problem: Find a minimal set of prime implicants, say $p_1, \ldots, p_k$, such that $(p_1 + \cdots + p_k) = 1$ iff $Y = 1$.

However, being interested in all 'logically irreducible and nonredundant combinations of attributes', as referred to in the above quotation, this minimization problem is of no specific importance. Indeed, one would need a complete list of all prime implicants. In the example of the subsection 'An Introductory Example', the solution of the minimization problem is $A.C + B.C'$, but there is a further prime implicant, $A.B$, not visible in the solution of the minimization problem but likewise interpretable as a non-reducible constellation of conditions.[13]

Moreover, in many applications the minimization problem has no unique solution but one can find several, or even a large number of, minimal sets of prime implicants whose addition is logically equivalent with $Y = 1$. (An example will be given in the next subsection.)

Data with Limited Diversity

So far it has been assumed that data are complete and do not exhibit contradictions so that they immediately determine a Boolean function with unrestricted domain. In many applications this will not be the case. In this subsection, I briefly discuss incomplete data still assuming that they do not exhibit contradictions.

To illustrate I use an example of Ragin (1987, p. 129) which is based on Stein Rokkan's data on divided working-class movements in Western Europe (Rokkan, 1970). The dependent variable is

Table 2.

C	*R*	*L*	*E*	*S*	
0	0	0	0	1	(Italy)
0	0	0	1	1	(France)
0	0	1	1	1	(Spain)
0	1	0	0	0	(Belgium, Luxembourg)
0	1	1	0	0	(Austria, Ireland)
1	0	0	0	1	(Finland, Iceland, Norway)
1	0	0	1	0	(Denmark, Sweden)
1	0	1	1	0	(Great Britain)
1	1	0	1	0	(Netherlands, Switzerland)
1	1	1	0	1	(Germany)

S ('major split in working-class movement provoked by Russian revolution'). Independent variables are: *C* ('national church vs. state allied to Roman Catholic church'), *R* ('significant Roman Catholic population and Roman Catholic participation in mass education'), *L* ('state protection of landed interests'), and *E* ('early state'). Table 2 shows the data.

For 6 of 16 constellations of the independent variables data are missing.[14] How then construct a Boolean function? There are different possibilities.

(a) A straightforward option is to restrict the domain of the function to the observed constellations of the independent variables. In the literature, this option is often called the 'don't care' approach. In the example one gets the prime implicants:

$$C.R.L, C'.R', C'.E, R'.E', C.E'$$

and a unique solution of the minimization problem described in the subsection 'Why Minimal Function Descriptions?':

$$S = C'.R' + C.E'$$

(b) Another possibility is to assume $S = 0$ for the missing constellations. One then gets four prime implicants:

$$C.R.L.E', C'.R'.L', C'.R'.E, R'.L'.E'$$

and again a unique solution of the minimization problem:

$$S = C.R.L.E' + C'.R'.E + R'.L'.E'$$

(c) Conversely, one can assume $S = 1$ for the missing constellations. One then finds *R.L.E.* in addition to the prime implicants found in (a), and there are now four solutions of the minimization problem:

$$S = C'.E + C.E' + R.L.E + C'.R'$$

$$S = C'.E + C.E' + R.L.E + R'.E'$$

$$S = C'.E + C.E' + C.R.L + C'.R'$$

$$S = C'.E + C.E' + C.R.L + R'.E'$$

It has been argued that, being interested in conditions of positive outcomes ($S = 1$), a conservative strategy would be (b).[15] This is true in the following sense: each of the prime implicants found with strategy (a) is logically implied by at least one of the prime implicants found with strategy (b). However, correctly understood the most prudent strategy is (a) because this strategy *restricts the domain* of the function to be constructed. The function resulting from this approach is to be applied only to constellations of the independent variables which have been observed at least once.

Is QCA a Case-oriented Method?

I have tried to show that QCA can be understood in two different ways, either as a method of data description or as a method of model construction. Causal interpretations presuppose the construction of a model. Before this will be discussed in the next section, I briefly criticize Ragin's claim that QCA is a case-oriented method.[16] The remarkable point is that the method in its technical sense (as a method of model construction) in no way depends on separate investigations and explanations of the cases that finally become part of a QCA data set.[17] In fact, the method *begins* with the definition of statistical (or modal) variables to provide a conceptual framework for subsequent investigations. In this respect, as explicitly said by Griffin and Ragin (1994, p. 10), there is no essential difference between QCA and other variable-oriented approaches:

Despite the greater interpretive thrust of QCA, the epistemological foundations of the analytic determination of causal relationships are similar in QCA and statistical analysis. Both are general and comparative in logical operation (Griffin, 1992). That is, both QCA and statistical analyses look for patterns across cases. Cases are considered as discrete, multiple instances of more general phenomena, thus permitting their aggregation into a set for the purpose of analysis. Attributes of the cases are then logically or statistically compared across all cases to form inductive causal (logical or statistical) generalizations that are then used deductively to explain or subsume the outcome of each individual case.

Instead of cases (as considered in case studies) the method considers configurations of variables. While cases may serve to illustrate these configurations and provide inspiration for the selection and definition of variables, the goal is to find functional relationships between variables. However, exactly this is often considered a hallmark of 'quantitative methods':

Quantitative methods focus directly on relationships among variables, especially the effects of causal or *independent* variables on outcome or *dependent* variables (Ragin, 1994, p. 145) [emphasis in original].

Remaining differences only concern the kind of functions used to connect independent and dependent variables. While conventional statistical approaches employ stochastic functions, QCA employs deterministic functions. This will be further discussed in the 'Deterministic and Stochastic Models' section.

Causal Interpretations

Causes as Sufficient Conditions

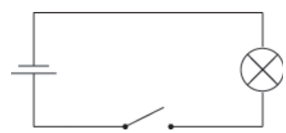
QCA is widely considered as a method of causal analysis.[18] Since the expression 'causal' is used in many different meanings,[19] it is important to understand its specific meaning in the context of the QCA approach. The basic idea is to think of the cause of some phenomenon as the (complex of) conditions on which the occurrence of the phenomenon (in some sense) depends. This approach to understand 'causality' was made popular by John St Mill in the 19th century.[20] For Mill, the cause of a phenomenon is 'the sum total of the conditions, positive or negative taken together; the whole of the contingencies of every description, which being realized, the consequent invariably follows' (Mill, 1879, Vol. I, p. 383). The causal interpretation of the QCA approach follows Mill in mainly three points:[21]

(a) Causes are conceptualized in terms of sufficient conditions.
(b) Causes are understood as *combinations* of conditions which together are sufficient for the occurrence of some phenomenon. This has been called 'conjunctural causation' by Ragin.
(c) It is recognized that different combinations of conditions might be sufficient for the occurrence of some phenomenon ('multiple conjunctural causation').[22]

These three ideas directly relate to a causal interpretation of the Boolean functions produced by the QCA approach. Assume a Boolean function $Y = f(X_1, \ldots, X_m)$. Then, (i) every prime implicant is a cause (in the sense of a sufficient condition) for $Y = 1$; (ii) prime implicants often relate to two or more of the independent variables; (iii) one often finds two or more different prime implicants. For example, there are three prime implicants, each consisting of two variables, in the example introduced at the beginning of the 'Understanding the QCA Approach' section: $A.C$, $B.C'$, and $A.B$. Using the notion of 'coverage' proposed by Ragin (2008, pp. 54–68), $A.C$ has coverage 9/16,[23] $B.C'$ has coverage 7/16, and $A.B$ has coverage 5/16.

Modal Interpretations Require Models

It is important to understand that talk of sufficient conditions can only be explicated by referring to functional models.[24] As an example, consider the following setup:



A battery and a bulb are connected by a circuit that can be closed or opened by a switch. Depending on the position of the switch the bulb gives light or not. One can easily construct a functional model using three variables:[25] $\ddot{Y}$ records whether the bulb gives light ($\ddot{Y} = 1$) or not ($\ddot{Y} = 0$), $\ddot{X}$ records whether the switch is closed ($\ddot{X} = 1$) or not ($\ddot{X} = 0$), and $\ddot{Z}$ records whether the battery provides power ($\ddot{Z} = 1$) or not ($\ddot{Z} = 0$). Taking $\ddot{Y}$ as endogenous and $\ddot{X}$ and $\ddot{Z}$ as exogenous variables, there is a simple Boolean relationship: $\ddot{Y} = \ddot{X}.\ddot{Z}$. The bulb gives light if the battery provides power and the switch is closed; in all other cases the bulb is off.

One might therefore say: that the battery provides power and the switch is closed, taken together, is a sufficient condition for the bulb to give light. However, it is obvious that this is only a sufficient condition with respect to the model. In any concrete situation where the model is applicable it is quite possible that the bulb is off although the battery provides power and the switch is closed. This is simply due to the fact that a model can only consider a limited (actually very small) number of conditions by explicitly defined variables.

A Deterministic Understanding of Causation?

Mill certainly had a deterministic understanding of causation: 'We may define [...] the cause of a phenomenon, to be the antecedent, or the concurrence of antecedents, on which it is invariably and *unconditionally* consequent.' (Mill, 1879, Vol. I, p. 392; emphasis in original). Such a deterministic understanding seems to be implied by the notion of causes as sufficient conditions,[26] and therefore is often also associated with the QCA approach, in particular by many of its critics.[27] However, one needs to distinguish:

(a) using deterministic functional models for the formulation of causal rules; and
(b) to believe that there are deterministic connections between empirically identifiable 'antecedents' and 'consequents'.

QCA implies (a), but is independent of (b). For example, using the deterministic model introduced in the previous subsection does not require to believe in deterministic connections between any states (of affairs) definable by referring to batteries, switches, and bulbs. It is well possible to criticize (b) as being an obscure metaphysical idea,[28] but this does not immediately imply a critique of the QCA approach. The relevant question is not, as put by King, Keohane and Verba (1994, p. 89), whether 'the world, at least as we know it, is probabilistic rather than deterministic', but rather, what kind of models ought to be used. In general, depending on the application, both deterministic and stochastic models can be useful. (This question will be further discussed in the 'Deterministic and Stochastic Models' section.)

Ambiguous Talk of Necessary Conditions

Complementary to sufficient conditions one can think of necessary conditions. Ragin (1987, p. 99) gave the following explanation: 'A cause is defined as necessary if it must be present for a certain outcome to occur. A cause is defined as sufficient if by itself it can produce a certain outcome.' Of course, both notions can be given a clear meaning only in the context of a functional model. Even then it might be unclear what is meant by the word 'cause'. If there are two or more exogenous variables, the word may refer either to values of single variables or to combined values of several variables.

In the context of Boolean functional models, causes in the sense of sufficient conditions are conceptually equivalent with prime implicants and most often simultaneously refer to two or more variables. This immediately implies: if there is only a single prime implicant it is also a necessary cause; otherwise necessary causes do not exist.

Alternatively, one can refer to single variables. I then speak of *causal factors* to avoid confusion with 'cause' in the sense of prime implicants.[29] It then follows that a causal factor is a cause just in the case that it is also a prime implicant. This might or might not be the case. For example, the model introduced in the subsection 'Modal Interpretations Require Models' contains two causal factors, $\ddot{X}$ and $\ddot{Z}$, but neither is a cause; the only cause of $\ddot{Y} = 1$ is the prime implicant $\ddot{X}.\ddot{Z} = 1$.

Referring to causal factors provides further possibilities to think of necessary conditions. Obviously, in the just mentioned example, both $\ddot{X} = 1$ and $\ddot{Z} = 1$ are necessary conditions for the bulb to give light. But consider the example discussed in the subsection 'An Introductory Example' of the 'Understanding the QCA approach' section: neither $\ddot{A} = 1$ nor $\ddot{B} = 1$ nor $\ddot{C} = 1$ is necessary for $\ddot{S} = 1$. So there are no necessary causal factors in this example. One might well say, however, that both $\ddot{A} = 1$ and $\ddot{C} = 1$ are necessary parts of the cause $\ddot{A}.\ddot{C} = 1$. This motivates the following notion proposed by Mackie (1965, 1980): a causal factor is an *INUS condition* if it is a nonredundant part of a cause (in the sense of a sufficient condition); the cause might or might not be necessary for the specified outcome.[30]

Static and Dynamic Notions of Cause

Mackie's idea was to offer 'INUS condition' as an explication of 'cause'. The proposal is interesting for several reasons. First, it respects the ordinary talk in which causes do not mean sufficient conditions but quite specific circumstances having some identifiable consequences. Second, it contributes to an explication of this ordinary understanding by pointing to the fact that the ways in which causal factors have consequences most often depend on circumstances.

A further interesting point becomes apparent when recognizing that 'cause' in the sense of 'sufficient condition' is basically a static notion. In contrast, ordinary talk of causation is most often dynamic, in the sense that causal connections are taken to exist between changes (events); for example: the bell-push *was pressed*, and as a causal consequence the bell *began to ring*. A dynamic version of INUS conditions can capture this intuition. I propose the following definition which refers to a function $Y = f(X_1, \ldots, X_m)$ of some functional model:

Given a covariate context of $\ddot{X}_i$,[31] say $\ddot{Z} = z$, a change $\Delta(x', x'')$ in the variable $\ddot{X}_i$ is a *functional cause* of a change $\Delta(y', y'')$ in the variable $\ddot{Y}$ iff $\ddot{Y} = y'$ can be derived from $\ddot{X}_i = x'$ and $\ddot{Z} = z$, and $\ddot{Y} = y''$ can be derived from $\ddot{X}_i = x''$ and $\ddot{Z} = z$.

To illustrate with the example of the subsection 'Modal Interpretations Require Models': in the covariate context $\ddot{Z} = 1$ (but not in the covariate context $\ddot{Z} = 0$), a change $\Delta(0,1)$ in $\ddot{X}$ functionally causes a change $\Delta(0,1)$ in $\ddot{Y}$.

The definition obviously does not require $Y = f(X_1, \ldots, X_m)$ to be a Boolean function; it can be used with any kind of deterministic function (defined in some functional model) and can also be extended to stochastic functions.[32]

A New Approach to Incomplete Data

The notion of functional cause also suggests a new approach to the problem of incomplete truth-tables (see 'Data with Limited Diversity' in the 'Understanding the QCA Approach' section). The idea is to find, for each of the independent variables in turn, all covariate contexts in which a change $\Delta(0,1)$ in the independent variable has a *positive effect* [implies $\Delta(0,1)$ in the dependent variable], or a *negative effect* [implies $\Delta(1,0)$ in the dependent variable], or *no effect*, or possible effects are *unknown* (due to missing data).

To illustrate, I use the example of 'Data with Limited Diversity' in the 'Understanding the QCA Approach' section. Table 3 shows what can be said about effects in this example.

The table not only shows what is not known due to missing data, but it also shows what can faithfully be said with the given data. For example, the only variable that has a positive effect (in a specific context) is *C* ('national church vs. state allied to Roman Catholic church'). Adding further data may change some of the entries in the table, but will not lead to contradictory results.

Table 3.

	Pos. effect	Neg. effect	No effect	Unknown
C	$R.L.E'$	$R'.E$	$R'.L'.E'$	$R'.L.E'$, $R.L'$, $R.E$
R	–	$C'.L'.E'$	$C'.L'.E'$	$C'.E$, $C.E'$, L
L	–	–	$C'.R.E'$, $R'.E$	$R'.E'$, $C.E'$, $R.E$, $C.R$
E	–	$C.R'.L'$	$C.R'.L'$	L, R

Deterministic and Stochastic Models

Data with Contradicting Outcomes

So far it has been assumed that the given data are immediately compatible with the assumption of a deterministic function. Real data, however, often exhibit so-called *contradictions*, meaning cases that share the same constellation of the independent variables but have different values in the dependent variable.

Given such data, some way of coping with the contradictions is required. There are different possibilities.

(a) One possibility is to search for additional independent variables that might be used to get rid of the contradictions.[33]

(b) Another possibility would be to use a subset of the cases that do not exhibit contradictions, and consider the remaining cases as 'exceptions' resulting from conditions not explicitly represented in the model. A version of this strategy would be to omit all conflicting cases.

(c) Finally, one can try to construct a stochastic instead of a deterministic functional model.

Ragin has mainly proposed strategies (a) and (b).[34] Here I want to discuss strategy (c); how one can understand stochastic functional models, and in which sense can they provide an alternative to deterministic models.

Deterministic and Stochastic Functions

Assume two deterministic variables, say $\ddot{X}$ and $\ddot{Y}$, having ranges $\mathcal{X}$ and $\mathcal{Y}$, respectively. A deterministic function $f: \mathcal{X} \rightarrow \mathcal{Y}$ assigns to each value $x \in \mathcal{X}$ a unique value $f(x) \in \mathcal{Y}$. In contrast, a *stochastic function* assigns to each value $x \in \mathcal{X}$ a probability distribution over $\mathcal{Y}$. Thus, instead of $\ddot{Y}$, one has to consider a *stochastic variable*, $\dot{Y}$, that can be used for probabilistic statements. A stochastic function may then be written as

$$x \rightarrow \Pr[\dot{Y} | \ddot{X} = x] \quad (3)$$

To each value $x \in \mathcal{X}$ the function assigns a conditional probability distribution of $\dot{Y}$, given that $\ddot{X} = x$.[35] In order to distinguish stochastic from deterministic connections between variables, stochastic functions will be graphically depicted as $\ddot{X} \twoheadrightarrow \dot{Y}$. Note also that $\Pr[\dot{Y} \mid \ddot{X} = x]$ is a probability distribution and not just a number. However, if $\dot{Y}$ is a binary variable with range {0,1}, it obviously suffices to consider the stochastic function $x \rightarrow \Pr(\dot{Y} = 1 | \ddot{X} = x)$ instead of (3).

Stochastic Models for QCA Applications

A stochastic functional model is a functional model that contains at least one endogenous stochastic variable and consequently at least one stochastic function. Interpretations depend on the application context. Here, we are interested in using such models for QCA applications. To illustrate, I use the example of 'Data with Limited Diversity' in the 'Understanding the QCA Approach' section. Assume that variable C has not been observed. The available data are then given as shown in Table 4.

A stochastic model would be $(\ddot{R}, \ddot{L}, \ddot{E}) \twoheadrightarrow \dot{S}$, using a stochastic function

$$(r, l, e) \rightarrow \Pr(\dot{S} = 1 | \ddot{R} = r, \ddot{L} = l, \ddot{E} = e) \qquad (4)$$

The observed frequencies in the last column of Table 4 might be used to estimate the conditional probabilities. However, since the model has not the task to describe the observed data, the question remains how to interpret the stochastic function.

Interpretation of Stochastic Functions

How to interpret the stochastic functions of a functional model depends primarily on the kind of generalization that the model is intended to provide (see 'Descriptive and Model Generalizations' in the 'Understanding the QCA Approach' section). If a descriptive generalization is intended, the model theoretically relates to a (specified) finite population, and the probabilities postulated by the model can be interpreted as frequencies with respect to the population. The probabilities are then conceptually analogous to the frequencies observed in the given sample, and the estimation problem has a definite meaning. In the example just mentioned, one would think of the 16 countries as being a sample of some population of countries and understand the model as intending statements about conditional frequencies in that population.

The situation is quite different if the model is intended to serve a modal generalization. It is not possible, then, to explicate the probabilities used in the formulation of the model by referring to frequencies in some population. There is, however, an alternative interpretation that becomes apparent when the functions formulated by a functional model are understood as rules. The probabilities used in the formulation of a stochastic function can then be interpreted as expressing the uncertainty of the outcome referred to by the function. For example, following this interpretation, the function (4) does not make a statement about some population of countries, but formulates a rule; and the formulation uses conditional probabilities to indicate the amount of uncertainty in the rule's predictions.

It is remarkable that stochastic functions do not require assumptions about any sources of the uncertainty which they express. One might well believe that some part of the uncertainty is due to missing variables. But the only way to argue for such a belief would be to find a better model that would allow better predictions.

Using Stochastic Functions for QCA

I have tried to show that, if understood as rules, deterministic and stochastic functions can be used quite analogously. The main difference concerns the expression of uncertainty. Deterministic functions do not deny uncertainty (or indeterminacy) but refer this problem to the application context. In contrast, stochastic functions make explicit statements about (some of) the uncertainty to be expected when used for predictions.

This confirms the conclusion of 'A Deterministic Understanding of Causation?' in the 'Causal Interpretations' section, that no fundamental choice between deterministic and stochastic models is required. In fact, without a theory providing ideas for a sufficient set of exogenous variables to avoid contradictory observations, it might well be preferable to use stochastic functions in order to represent the indeterminacy found in the given data.

An interesting consequence concerns the causal interpretation. Using stochastic functions, the static

Table 4.

			Cases with		
R	***L***	***E***	***S* = 0**	***S* = 1**	***P(S* = 1|*R, L, E)***
0	0	0	0	4	1
0	0	1	2	1	1/3
0	1	1	1	1	1/2
1	0	0	2	0	0
1	0	1	2	0	0
1	1	0	2	1	1/3

view that thinks of causes as sufficient (and/or necessary) conditions no longer makes sense. More reasonable is a stochastic version of the definition of functional causality given in ‘Static and Dynamic Notions of Cause’ of the ‘Causal Interpretions’ section. If all variables are binary, one can again follow the approach proposed in ‘A New Approach to Incomplete Data’, namely to find, for each of the independent variables in turn, all covariate contexts in which a change $\Delta(0,1)$ in the independent variables has

(a) positive effect: $\Pr(\dot{Y} = 1|\ddot{X} = 1, \ddot{Z} = z) > \Pr(\dot{Y} = 1|\ddot{X} = 0, \ddot{Z} = z)$,
(b) a negative effect: $\Pr(\dot{Y} = 1|\ddot{X} = 1, \ddot{Z} = z) < \Pr(\dot{Y} = 1|\ddot{X} = 0, \ddot{Z} = z)$,
(c) no effect: $\Pr(\dot{Y} = 1|\ddot{X} = 1, \ddot{Z} = z) = \Pr(\dot{Y} = 1|\ddot{X} = 0, \ddot{Z} = z)$, or
(d) possible effects are unknown (due to missing data).

Here $\ddot{X}$ is used for the causal factor, and $\ddot{Z} = z$ represents a covariate context. Similar to the search for prime implicants of Boolean functions, one might search for minimal descriptions of covariate contexts that are required to determine the different possibilities.[36]

Conclusions

The article has shown that QCA can be understood in two quite different ways. On the one hand, it can be understood as a method for the construction of Boolean functions that describe a given set of data. On the other hand, it can be understood as a method for the construction of functional models. The distinction is important because it is closely linked to the question whether, and in which sense, QCA can be used for the (hypothetical) formulation of causal relationships. The article has argued that this requires to consider QCA as a method for the construction of functional models.

This understanding, moreover, opens the opportunity to locate QCA in the broader framework of functional models that includes deterministic as well as stochastic versions. QCA is most often understood as a method for the construction of deterministic models which allow one to think of causes in terms of necessary and/or sufficient conditions. However, when data exhibit contradicting outcomes it could be a useful strategy to consider stochastic models instead of modifying the data in order to make them compatible with a deterministic model. It will no longer be possible, then, to think of causes in terms of necessary and/or sufficient conditions. However, the article briefly hints to a notion of functional causality that can be defined in quite similar ways both for deterministic and stochastic models.

Notes

1. For an introduction and overview see the textbook edited by Rihoux and Ragin (2009). Another introductory textbook is Schneider and Wagemann (2007). These textbooks also provide references to a broad range of applications.
2. In addition to the textbooks cited in Note 1, see Yamasaki and Rihoux (2009). Extensive information is also provided by the COMPASSS Web site (www.compasss.org).
3. These notations are used: $X.Y{=}1$ iff $X{=}1$ and $Y{=}1$; $X + Y{=}1$ iff $X{=}1$ or $Y{=}1$; $X' = 0$ iff $X{=}1$. The logical multiplication (and) has priority over the logical addition (or). The expression ‘iff’ is short for ‘if and only if’.
4. The term *property space* is used here to denote any set of properties that can be used to characterize the elements of a reference set.
5. In this text, **R** denotes the set of real numbers.
6. X may consist of several components, e.g. $X = (A, B, C, S)$. Since the distinction between one- and multi-dimensional statistical variables is purely formal and has no substantive meaning, it will only be mentioned if components need to be distinguished.
7. P[X] will be taken as a function that provides, for each element or subset of $\mathcal{X}$, the corresponding proportion of cases in Ω. Referring to a specific value $x \in \mathcal{X}$, a standard notation would be $P(X = x)$, meaning the proportion of cases in Ω for which the variable X has the value x. For example, using the data in Table 1, one finds the proportion $P((A, B, C, S) = (1, 0, 1, 1)) = 6/38$.
8. In this text, the term ‘modal’ will be used to remind of possibilities (and probabilities). For a broader discussion of ‘modal thinking’, see White (1975).
9. The designation reflects the very broad usage of the term ‘causal’ in the scientific literature. It is certainly reasonable, and will be done below, to distinguish different understandings and definitions of causality.

10. A variable is called *endogenous* (or dependent) if its values depend on other variables defined in the model; otherwise it is called an *exogenous* (or independent) variable.
11. For a detailed discussion of deterministic and stochastic functional models, see Rohwer (2010).
12. Of course, like statistical variables, modal variables may consist of several components, e.g. $\ddot{X} = (\ddot{X}_1, \ldots, \ddot{X}_m)$ having the range $\mathcal{X} = \mathcal{X}_1 \times \cdots \times \mathcal{X}_m$ or some subset of this product.
13. See the remarks made by Ragin (1987, p. 98). For an illustration of the temptation to focus solely on a solution of the minimization problem, see Ragin et al. (2003, pp. 331–336).
14. Proposed by Ragin, this kind of missing data is called *limited diversity* in the QCA literature; see, e.g. Ragin (1987, p. 204), Ragin and Rihoux (2004a, p. 7), and Schneider and Wagemann (2007, p. 101). The missing constellations are often called *remainders*.
15. See Ragin and Rihoux (2004a, p. 7).
16. See, e.g. Ragin (1998).
17. I here follow Gerring's (2004) understanding of case study research. The argument, of course, does not exclude the possibility to use QCA in case study research.
18. See, e.g. Ragin (1987, 2000), Mahoney (2000, 2003, 2008), Rihoux (2006), Schneider and Wagemann (2007).
19. See, e.g. Cartwright (2004).
20. The relevant source is Mill's *System of Logic* which first appeared in 1843. I refer to the 10th edition of 1879.
21. See Ragin (2000, pp. 99–104).
22. See already Mill (1879, Vol. I, p. 504); see also Mackie (1965: 61).
23. There are 16 cases with $S = 1$, and in nine of these cases is $A.C = 1$.
24. Ragin (1987, p. 99) has made a similar point: 'Neither necessity nor sufficiency exists independently of theories that propose causes.'
25. Since the model is deterministic, its variables are marked by two points.
26. See, e.g. Mahoney (2008).
27. For example, Lieberson (1992, 1994, 1997), Goldthorpe (2000, p. 50), and King, Keohane and Verba (1994, pp. 87–89). See also the discussion of 'deterministic explanations' by Mahoney (2003).
28. The philosophical question eventually concerns whether, outside the realm of analytic truths, the idea of 'a totality' of sufficient conditions can be given any clear meaning.
29. It is assumed that the expression 'causal factor' refers to a (binary) variable having a specified value.
30. INUS is an abbreviation of 'insufficient but nonredundant part of an unnecessary but sufficient condition'.
31. A covariate context of a variable $\ddot{X}_i$ is any selection of variables from $\ddot{X}_1, \ldots, \ddot{X}_m$ that does not contain $\ddot{X}_i$ which are given specified values.
32. For a detailed discussion of this notion of functional causality, see Rohwer (2010).
33. Instead of searching for additional variables, it sometimes might be possible to get rid of contradictions by redefining already included variables (e.g. by changing cut-off thresholds for dichotomization). This possibility was suggested by an anonymous reviewer.
34. See Ragin (1987, pp. 113–118; 1994, p. 120), Ragin and Rihoux (2004b, p. 23), and Ragin (2008, pp. 27–28).
35. The notation $\Pr[\dot{Y}]$ is used for the probability distribution of the stochastic variable $\dot{Y}$, formally analogous to the notation $P[Y]$ for the frequency distributions of a statistical variable Y; see Note 7. Correspondingly, $\Pr(\dot{Y} = y)$ is used to denote the probability of $\dot{Y} = y$ and distinguished from $P(Y = y)$, that is, the frequency of $Y = y$ in some specified reference set.
36. For a similar approach, called 'logic regression', see Ruczinski, Kooperberg and LeBlanc (2003).

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