A Temporal View of Time for Event-History Analysis

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Introduction

Reference to time is a basic prerequisite for event-history analysis and its variety of statistical models. In order to set up specific statistical models it does not suffice to employ ordinary language expressions but one needs a mathematical representation of a time axis. Unfortunately, standard mathematics only offers a choice between a discrete representation, based on natural numbers, or a continuous representation, based on real numbers. This paper discusses some limitations of both forms of representing time and considers an alternative view that represents the occurrence of events by time intervals.

1 Events

Being children of a specific history, we have learned to make temporal references by using clocks and calendars and to think of time as a linear time axis organized by these tools. But leaving aside, for the moment, clocks and calendars, what enables us to speak about time? One possible approach that I will follow here begins with the notion of event. An event is something that occurs. The notion is extremely general and therefore quite difficult to make precise. However, for the present purpose, it seems possible to neglect philosophical discussion and simply begin with a common sense view of events.[1] The following four points seem to be essential.

- The occurrence of an event always involves one or more objects whose properties change in some way when the event is occurring.
- Each event has some finite temporal duration.
- For many events one can say that one event occurred earlier than another event.
- Events can be characterized, and classified, by using the linguistic construct of kinds of events.

Using these assumptions it seems, first of all, important to distinguish between *events* and *kinds of events*. An event is something unique, for example, the event that two specific individuals become married. A corresponding kind of event would be 'to become married'. The same kind

[1] For related philosophical discussion see Hacker 1982, and Lombard 1986.

of event can occur several times. Therefore, characterizing an event as being of a certain kind does not give a unique description. Furthermore, a single event does not necessarily belong to only a single kind of event. Most often one can characterize a single event as an occurrence of several different kinds of events. For example, an event that is a marriage can also be a first marriage of two people.

While common language clearly distinguishes between objects and events, one might well think of a certain correspondence between, on the one hand, objects and their properties, and on the other hand, events and kinds of events. This has led some authors, e.g., Brand (1982), to think of objects and events as being ontologically similar. Whether or not this might be sensible, I shall assume that talking of events always implies a reference to objects and intends to capture the notion of change in the properties, or behavior, of objects. However, it seems not necessary to be rigorous on this point. It will suffice to require that it should be possible to associate, with each event, *some* objects that are involved in the event. In general, these objects need not be individuals in the sense of behavioral units.

Following the common sense view of events it also seems obvious that events occur "in time". In fact, the notion of event provides one basic understanding of time. It seems sensible, therefore, to assume that one can associate with each event, e, a certain location in time, $t(e)$. $t(e)$ will be called the *t-location* of the event e. While a strict definition cannot be given it seems important to think of t-locations *not* as being "time points". Quite to the contrary, one of the most basic facts about events is that each event has a certain temporal duration. This is not only obvious when we think of standard examples of events, but seems logically implied if we think of events in terms of change. Change always needs some amount of time. This then has a very important further implication: only when an event *has occurred* and, consequently, when it has become a fact belonging to past history, can we say that the event has, in fact, occurred. We cannot say this *while* the event is occurring.[2]

[2]Thinking of actions as particular types of events, this implication has been described by Danto (1985, p. 284) as follows: "Not knowing how our actions will be seen from the vantage point of history, we to that degree lack control over the present. If there is such a thing as inevitability in history, it is not so much due to social processes moving forward under their own steam and in accordance with their own natures, as it is to the fact that by the time it is clear what we have done, it is too late to do anything about it."

That one thinks of events in terms of change is quite essential for the common sense view of events that I try to follow here. Without a change nothing occurs. Fortunately, one need not be very specific about what kind of changes occur. It suffices to require that one can think of at least one object that, in some sense, changes its properties. Whether these changes occur "continuously" or "instantaneously" is quite unimportant as long as we require that the event has some temporal duration. The event is then defined by what happened during this amount of time and must be taken as a whole. One might be able to give a description of the event in terms of smaller sub-events. But these will then simply be different events. In this sense is an event semantically indivisible.

Finally, it is important that one can often say of two events that one occurred earlier than the other. Of course, this cannot always be said. One event may occur while another is occurring. However, there are many clear examples where we have no difficulties to say that one event occurred earlier than another one. I shall therefore assume that the following partial order relations are available when talking about events (e and e' are used to denote events).

$$e \preccurlyeq e' \quad \text{meaning: } e' \text{ begins not earlier than } e$$
$$e \lhd e' \quad \text{meaning: } e' \text{ begins not before } e \text{ is finished}$$
$$e \sqsubseteq e' \quad \text{meaning: } e \text{ occurs while } e' \text{ occurs}$$

All three relations are only partial order relations. Nevertheless, they can be used to define corresponding relations between the t-locations of events. I will use the same symbols.

$$t(e) \preccurlyeq t(e') \iff e \preccurlyeq e'$$
$$t(e) \lhd t(e') \iff e \lhd e'$$
$$t(e) \sqsubseteq t(e') \iff e \sqsubseteq e'$$

It will be said that a set of events is equipped with a *qualitatively ordered time axis* if these three relations are available.

As an illustration consider the four events in Figure 1.1 where one can find the following order relations:

$$e_1 \preccurlyeq e_2,\ e_1 \preccurlyeq e_3,\ e_1 \preccurlyeq e_4,\ e_2 \preccurlyeq e_3,\ e_2 \preccurlyeq e_4,\ e_3 \preccurlyeq e_4$$
$$e_1 \lhd e_3,\ e_1 \lhd e_3,\ e_2 \lhd e_3,\ e_2 \lhd e_3$$
$$e_2 \sqsubseteq e_1$$

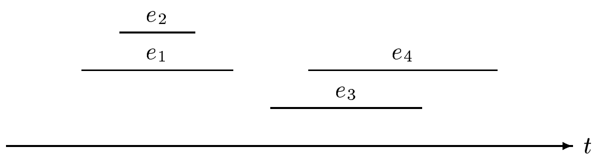


Fig. 1.1 Illustration of order relations between four events on a qualitatively ordered time axis.

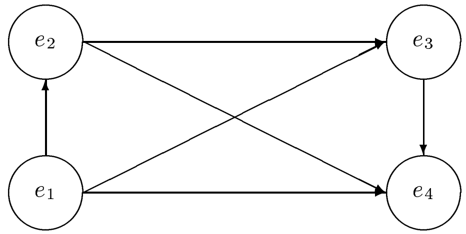


Fig. 1.2 Graph illustration of '$\preccurlyeq$' relation between the four events shown in Figure 1.1.

Of course, on a qualitatively ordered time axis, the lengths of the lines used in Figure 1.1 to represent events do not have a quantitative meaning in terms of duration. This becomes clear if one represents the order relations between events by means of a directed graph. This is illustrated in Figure 1.2 where the arcs represent the '$\preccurlyeq$' relation between the events.

Composing events. Our language is quite flexible to compose two (or more) events into larger events. Think, for example, of clock ticks as elementary events. It seems quite possible to think also of two or more successive clock ticks as events. To capture this idea into a formal language, one can introduce a binary operator, '$\sqcup$', that allows to create (linguistically) new events. The rule is: If e and e' are two events then also is $e \sqcup e'$ an event. Events created by using the operator $\sqcup$ will be called *composed events*. Thinking in terms of a class of events, one can assume that the class is closed with respect to $\sqcup$. In any case, this can be assumed consistently by extending the time order relations defined

above for composed events in the following way:

$$
\begin{aligned}
e \sqcup e' \preccurlyeq e'' &\iff e \preccurlyeq e'' \text{ or } e' \preccurlyeq e'' \\
e \sqcup e' \lhd e'' &\iff e \lhd e'' \text{ and } e' \lhd e'' \\
e \sqcup e' \sqsubseteq e'' &\iff e \sqsubseteq e'' \text{ and } e' \sqsubseteq e''
\end{aligned}
$$

This also allows to introduce the notion of elementary event. A possible definition would be that an event, say e, is an *elementary event* if there is no other event, e', such that $e' \sqsubseteq e$. Using this definition, one conceives of elementary events as not being divisible into smaller event units.

It might seem questionable whether elementary events do exist. When describing an event it often seems possible to give a description in terms of smaller and smaller sub-events, without definitive limit. However, we are not concerned here with the ontological status of events. Regardless of whether it is possible to give *descriptions* of events in terms of smaller sub-events, when talking about events one cannot avoid to assume *some* 'universe of discourse' that provides the necessary linguistic tools. This might be used to justify the assumption that the number of events that can be meaningfully assumed to have occurred in a finite period of time is itself finite. Given this assumption the existence of elementary events is an immediate consequence.

Interestingly, it seems not possible to define a converse operation, '$\sqcap$', using the interpretation that $e \sqcap e'$ occurs while e and e' are occurring. The reason is that we should be able to say that an event has, in fact, occurred as soon as the event no longer occurs. But this condition will in general not hold for $e \sqcap e'$ because one can only say that e *and* e' occurred when both are over. There is, therefore, no obvious way to define an algebra of events.

2 Duration

If an event, e, occurs while another event, e', is occurring ($e \sqsubseteq e'$), one can sensibly say that the duration of e is not longer than the duration of e'. This allows a partial ordering of events with respect to duration and can be used as a starting point for a quantitative concept of duration.

a) In order to measure the duration of an event, e, we count the number of pairwise not overlapping events, e', such that $e' \sqsubseteq e$.[3] The

[3] It will be said that two events, e' and e'', do not overlap if $e' \lhd e''$ or $e'' \lhd e'$.

maximal number of those events can be used as a discrete measure for the duration of e having t-locations as units. As an implication, all elementary events will then have a unit duration.

b) In the same way one can measure the duration between two events, say e and e'. Again, simply determine the maximal number of pairwise not overlapping events, e'', such that $e \lhd e'' \lhd e'$. If such an event cannot be found I shall say that e' *immediately follows* e.[4]

These definitions make duration dependent on the number of events that can be identified in a given universe of discourse. The obvious way to cope with this problem is to enlarge the number of events that can be used to measure duration. This is done by using clocks. Defined in abstract terms, a clock is simply a device that creates sequences of (short) events. Then, if a clock is available when an event occurs, its duration can be measured by counting the clock ticks that occur while the event is occurring.

Let e be the event whose duration is to be measured and let c_n denote an event composed of n clock ticks. One might then be able to find a number, n, such that

$$t(c_n) \sqsubseteq t(e) \sqsubseteq t(c_{n+1})$$

This will allow to say that the duration of event e is between n and $n+1$ clock ticks.

Many different kinds of clocks have been invented,[5] and this has led to the difficult question how to compare different clocks with respect to accuracy. Fortunately, we are not concerned here with the problem of how to construct good clocks. We shall certainly use the clocks that are commonly used in daily life to characterize, and coordinate, events. We are, however, concerned with the problem of what kind of numerical representation should be used for duration (whatever device is actually used for measuring). The fact that different clocks having different accuracy do exist becomes then an argument that we should find a numerical representation that is independent of any specific clock.

[4] In fact, we then do not have any reason to believe in a duration between e and e'. Leibniz (1985, p. 7) made this point by saying: “Ein grosser Unterschied zwischen Zeit und Linie: der Zwischenraum zwischen zwei Augenblicken, zwischen denen sich nichts befindet, kann auf keine Weise bestimmt werden und es kann nicht gesagt werden, wieviele Dinge dazwischen gesetzt werden können; [...] In der Zeit berühren sich daher die Momente zwischen denen sich nichts ereignet.”

[5] See, e.g., Borst 1990.

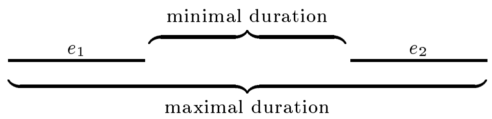
This suggests to use, for the numerical representation of duration, intervals of real numbers. Since duration is always positive a sensible choice is

$$\mathbf{R}_{]+]} := \{\,]\, a, b\,] \mid 0 \leq a < b, a, b \in \mathbf{R} \}$$

Clearly, this representation is not meant to imply that the duration of events, or between events, has sharp boundaries. Quite to the contrary, this representation is intended to allow for both conceptual and empirical indeterminacy.

Calendar time. Thinking of events we need to distinguish between t-locations and durations. The duration of an event tells us how long the event occurred compared with the occurrence of other events. The t-location of an event provides information about the location of the event in a set of events equipped with the partial orders, ‘$\preccurlyeq$’, ‘$\lhd$’, and ‘$\sqsubseteq$’. However, having available a concept of duration, one can also introduce quantitative statements about t-locations. The basic tool is the notion of *calendar*. In abstract terms, it can be defined by the specification of a base event and a concept of duration between events. This then allows to locate every event by providing information about the (positive or negative) duration between the event and the base event of the calendar.

To make this idea precise one needs a definition of duration between events. In principle, one can follow the approach already mentioned above. Then, having available a clock, the duration between two events, say e and e', can be measured by counting the number of non-overlapping clock events having a t-location between e and e'. However, this definition of duration between events is not fully satisfactory because the events also have a duration. This fact obviously creates some conceptual indeterminacy and it seems therefore preferable to proceed in terms of a minimal and maximal duration as follows.



This suggests to use again the set of positive real intervals, $\mathbf{R}_{]+]}$, now for the numerical representation of duration between events.

Summary. Each event refers in two different ways to time. First, it has an inherent duration. While this is, in the first instance, a purely qualitative notion it can be made measurable to allow a numerical representation by positive real intervals. It will be assumed, therefore, that one can associate with each event, e, a positive duration

$$\mathrm{dur}(e) \in \mathbf{R}_{]+]}$$

Of course, interpretation requires information about the kind of elementary events that have been used to measure duration. If all elementary events are of the same kind, as is normally the case when using clocks, one of these events (or a suitably defined composed event) provides a sensible unit of duration. In any case, it will most often be possible to assume that duration can be measured in some standard units like seconds, days, months, or years.

Secondly, one can associate with each event a t-location that provides information about the place of the event in the order of time. Again, in the first instance, this is a purely qualitative notion defined only with respect to three partial order relations between events. But, having available a measurable concept of duration, one can introduce a quantitative representation for the duration between events, again by using positive real intervals. Then, for each pair of events, e and e', one can use

$$\mathrm{dur}(e, e') \in \mathbf{R}_{]+]}$$

to represent the duration between the two events. Finally, one can introduce a calendar as a quantitative representation of t-locations. This means to specify a base event, $e^\dagger$, and then represent the t-location of any other event, say e, by the duration between e and $e^\dagger$. Then, if $e^\dagger \preccurlyeq e$, $\mathrm{dur}(e^\dagger, e)$ provides a quantitative representation of the t-location of e with respect to the calendar defined by $e^\dagger$.[6] So one finally can use a single numerical representation, $\mathbf{R}_{]+]}$, both for the duration and for the t-location of events.

Practical considerations. At present, the most commonly used calendar is the Gregorian where $e^\dagger$ is defined as birth of Christ.[7] For statistical calculations, this calendar has the disadvantage that days and weeks, and months and years, do not always stay in a fixed relation.

[6] If $e \preccurlyeq e^\dagger$, we can use the same approach by allowing for negative real intervals. However, for all practical applications we can chose the base event, $e^\dagger$, such that all other events occur later.

[7] See, e.g., Borst 1990. For additional information see Smith 1958, p. 651 et seq. Brüning (1985) has given an introduction to practical calculations with calendar dates.

As a consequence, it is not directly possible to calculate the duration between two dates if they are given in a (day/month/year) format. So it is often preferable to use alternative calendars.

a) When dealing with event data whose dates are given by year and month in the Gregorian calendar an often useful calendar is given by *century months*.[8] This calendar is defined by fixing $e^\dagger$ as the first identifiable event in January, 1900. An event that occurred in that month is then represented by the interval $]0, 1]$. In general, given a date in the Gregorian calendar by year (y) and month (m), the corresponding century month is

$$]\,t-1, t\,] \quad \text{with} \quad t = 12\,(y - 1900) + m$$

Conversely, given a century month $]\,t-1, t]$, one can recover the Gregorian year and month by the formulas[9]

$$\begin{aligned} y &= \lfloor (t-1)/12 \rfloor + 1900 \\ m &= (t-1)\,\%\,12 + 1 \end{aligned}$$

b) If dates are given in days another often used calendar is the Julian. Skipping over historical details, one can think of this calendar as defined by some specific day, say $e^\dagger$, in the Gregorian calendar. Moreover, this day can be chosen arbitrarily. The *Julian date* of an event, e, is then defined as the number of days (positive or negative) between e and $e^\dagger$. Practically, the Julian calendar offers formulas to convert Gregorian dates, given by (day, month, year), into Julian days, and vice versa.

3 Past and Future

A basic idea of descriptive statistics consists in the assumption that one can sensibly refer to a "given", in some sense existing, finite set of objects. Being concerned with applying statistical ideas to describe social reality, we most often refer to some collective of individuals in the sense of behavioral units. However, one of the most basic facts about individuals is that they do not exist forever. They come into being at

[8] See, for example, the data used in Blossfeld and Rohwer 1995, ch. 2.

[9] For any real number $x \in \mathbf{R}$, $\lfloor x \rfloor$ means the largest integer number not greater than x. % is used to denote the modulus operator: $x\,\%\,y$ means the remainder when dividing x by y.

some time and, sooner or later, they die. This suggests to associate with each individual two basic events:

$$b(\omega) := \text{birth of individual } \omega$$
$$d(\omega) := \text{death of individual } \omega$$

But in which sense do these events exist? Clearly, if we assume a collective consisting of existing individuals, say Ω, there must be a birth event for each individual. On the other hand, we can only speak of a death event when the event has occurred. But most often when referring to a collective, Ω, all or most of its members are still alive. The idea to associate with each member of Ω also a death event becomes, therefore, somewhat obscure.

A simple way out of this dilemma seems possible by introducing a distinction between *realized* and *possible* events. But while quite common in ordinary talk about events, the distinction is easily misleading. The point is that we can only speak of an event if it has, in fact, occurred. Otherwise we do not speak about facts but of possibilities, and we should clearly make a distinction between facts and possible states of affairs. Therefore, whenever we use the notion of 'possible event', we should be aware that we are not then referring to events. If somebody is still alive we can certainly speculate about his future fate and we can be sure that he will die somewhere in the future. But for the time being there simply is no death event that can be meaningfully associated with this person.

To account for this difference I shall follow here the common sense view in making a basic distinction between past and future. Then, if we speak of facts, in particular of events that have occurred, we always refer to past facts and past events. In a sense, this can be used to define the meaning of 'past'. On the other hand, the future is what is not already past and we then think of possibilities, possible events that might occur in the future.

Now, while this distinction between past and future is deeply rooted in human life, it is virtually absent in most physical theories, and theories that follow their view of time. The distinction between past and future is then seen as only an information problem, in the sense that we normally have less information about future events than about past events. But, apart from this information problem, there is no essential distinction between past and future.[10]

[10] Impressed by physical theories that conceive of time analogously to space, some philosophers have proposed that tensed expressions and tensed (modal) forms of reasoning should, in principle, be dismissed; see Smart 1963, ch. 7. This is also the central tenet of the "new theory of time" which is currently debated by many philosophers, see Oaklander and Smith 1994.

One therefore needs a decision which of the two roads to follow. In my view, the decision should be made dependent on what kind of knowledge one finally wants to establish and in this sense, is mainly independent of the quest for a philosophically satisfactory account of time. It seems quite possible to allow for different notions of time depending on what kind of theory is intended. Here I shall assume that we are interested in event-history analysis as a statistical approach that aims for a better understanding of social reality. Whatever models will finally be formulated by using this conceptual framework, I propose that it should be possible, for social actors, to integrate these models into their understanding of being actors in historical time. For this reason, I shall not follow the route of physical theories in assuming a given universe of events, but make the distinction between past and future a basic assumption of the conceptual framework.[11]

To follow this decision requires that, in principle, one always indicates a t-location that shows the temporal location from where one is referring to individuals and events assumed to exist in historical time. It is not, however, required that this t-location must be interpretable as the "present situation" of an observer. I find it preferable to avoid any usage of the word 'present' since it seems impossible to provide a definition independent of a self-referencing subject. Consequently, it will be tried to only use an artificial, or hypothetical, distinction between past and future. I shall assume that whenever one refers to some t-location this implies a *corresponding* distinction between past and future. Only events that occurred before this t-location can then be referred to as events that have, in fact, occurred.

Definition 3.1 To provide a shorthand notation, I shall use $\mathcal{E}_t$ to denote a set of events that have occurred before t. Here t is a positive real number and it will be assumed, by definition, that

$$\text{for all } e \in \mathcal{E}_t\colon\ t(e) \in \mathbf{R}_{]0,t]} := \{\,]\,a,b\,]\mid 0 \le a < b \le t,\, a,b \in \mathbf{R}\}$$

$\mathcal{E}_t$ can be interpreted as containing all events that, in a given context of discourse, one can reasonably think of having occurred in the time

[11] I take it as a secondary question whether this should be viewed as an ontological distinction. At least in a pragmatic sense, the distinction between past and future seems compatible also with the "new theory of time", see Mellor 1994.

interval $]\,0,t\,]$.[12] In addition, as mentioned above, it will be assumed that the number of events in $\mathcal{E}_t$ is finite.

Definition 3.2 Using this concept of a time-dependent set of events one can also introduce a tensed notion of collectives. Given some positive real number, t, the symbol Ω_t will be used to denote the finite set of individuals who have birth events in the interval $]\,0,t\,]$. Viewed as a function of t, Ω_t will be called a *temporal collective*.

4 Temporal View of Time

The notions introduced in the preceding sections allow for two complementary ways of referring to time. One can refer to $\mathcal{E}_t$ as a set of events that have occurred until t without immediately also referring to specific objects involved in these events. Or one can refer to temporal collectives, Ω_t, and then think of events as changes occurring in properties, or behavior, of its individual members. Both views can be used as a starting point for a construction of statistical models.[13] Here it remains to discuss an important distinction between two views of processes that develop in time, in certain respects parallel to the distinction between past and future, and also to the distinction between propositional and modal reasoning.

Retrospective view. One can take a *retrospective view* meaning to fix a certain time point, say t, and think of Ω_t and $\mathcal{E}_t$ as some, by t, completed set of facts. One then ignores the question of how these facts have come into being during the historical time leading to t. Instead one views all those facts as *simultaneously existing* from the point of view given by the fixed time point t.

Temporal view. Alternatively, one can take a *temporal*, or *historical*, view. If we follow this view, we do not place ourselves at a fixed t-location but, instead, try to follow the development of individuals and events in historical time. We cannot refer, then, to a fixed set of events but need to think of possibly occurring events. And correspondingly, we cannot refer to a fixed set of individuals but need a notion of temporal collective

[12] So one does not require that all events in $\mathcal{E}_t$ have been observed. Of course, in order to finally arrive at empirical statements, one needs information about observed events. But available information will always be limited and the distinction between theoretical concept and available information is therefore important.

[13] For a discussion of the complementarity of both views see Galton 1984, ch. 2.

whose members are coming and going. We then try to share, so to speak, our temporal location with the members of the collective to be described.

This leads to the question how to find a suitable formal representation of time that can be used for both, a representation of events and temporal collectives. There are different possibilities. Referring to collectives, the easiest way would be to fix a discrete sequence of t-locations and then to consider a corresponding sequence of collectives. But there is obviously a problem with this approach. The t-locations of the events that we want to describe may not and, in general, will not coincide with the t-locations used to define the discrete time axis.

Alternatively, one can try to consider a set of collectives, Ω_t, for all real numbers in some interval $]\,t_l,t_u\,]$. But also this approach has an obvious drawback. Because one can always find a new real number being in between of two given real numbers, one can no longer think of a sequence of t-locations. On the other hand, the notion of *sequence* certainly plays a fundamental role in our ordinary understanding of time as history. In order to cope with this dilemma I discuss an alternative that will be called a *temporal*, or *historical*, view of the time axis. The basic idea is to think of a partition of the time axis induced by the occurrence of events.

Definition 4.1 For any $t \in \mathbf{R}$, $t > 0$, let

$$\alpha(t) := \min\{\, t_l \mid \exists\, e \in \mathcal{E}_t : t(e) =]\,t_l,t_u\,]\,\}$$
$$\tau(t) := \max\{\, t_u \mid \exists\, e \in \mathcal{E}_t : t(e) =]\,t_l,t_u\,]\,\}$$

The partition of $]\,\alpha(t),\tau(t)\,]$, induced by the t-locations of events in $\mathcal{E}_t$, will be denoted by $\mathcal{L}_t$, its subsets will be called *τ-locations*.

Obviously, $\mathcal{L}_t$ is the most coarse partition of $]\,0,t\,]$ (excluding, if they exist, the intervals $]\,0,\alpha(t)\,]$ and $]\,\tau(t),t\,]$) such that the t-location of each event in $\mathcal{E}_t$ can be expressed as the union of one or more contiguous sub-intervals (τ-locations). For this reason, $\mathcal{L}_t$ will be called the *time axis induced by $\mathcal{E}_t$*.

Using now this concept of partition of time induced by events it becomes easy, at least in formal terms, to conceive of a temporal view of the time axis. The idea is to think of time not as evolving continuously but as created by the occurrence of events: each new event adds a certain duration to the time axis.[14] Since it was assumed that in each finite

[14] I do not think of this idea as a specific "philosophy of time", but simply use it as a starting point for a dynamic representation of events that is compatible with

interval only a finite number of events can occur, also each partition, $\mathcal{L}_t$, contains only a finite number of sub-intervals (τ-locations). So one arrives at a discrete notion of time, but 'discrete' in a very specific sense. In a temporal view, discreteness of time is not defined externally (from a position outside of historical time) but is taken as created by the occurrence of events. Then, given an event set $\mathcal{E}_t$, the smallest pieces of time are the τ-locations of the induced partition, $\mathcal{L}_t$.

Of course, the notion of τ-locations does not imply that there is always an event that occurs during a τ-location. In general, events can occur while other events are occurring and the occurrence of events can be overlapping (in the sense defined above). Nevertheless, a τ-location, say τ, can alway be characterized by referring to all events that contain τ while they are occurring. Some ambiguity may only arise when we approach "present" time, t, and some event occurs at t.[15] However, since we can only speak of an event when it *has occurred* and has become, therefore, a fact that occurred in the past, these events, by definition, do not belong to $\mathcal{E}_t$.

So one finally arrives at a notion of discrete time that avoids both drawbacks mentioned above. Its basic feature is that the elementary units of time, its τ-locations, are induced by the occurrence of events, not by an a priori fixed partition into time intervals. Nevertheless, one can speak of a *sequence* of τ-locations.

While the notion of τ-locations is fundamental for a temporal view of time, it is often more practical to use a complementary representation by recognizing the points in time when new information about events becomes available. I therefore add the following definition.

Definition 4.2 The set of real numbers, τ, such that there is an event in $\mathcal{E}_t$ having a t-location $]\,t_l, t_u\,]$ with $t_u = \tau$, will be denoted by $\mathcal{T}_t$ and its elements will be called *time points*.

In the same way as one can think of $\mathcal{L}_t$ as a sequence of τ-locations one can think of $\mathcal{T}_t$ as a sequence of time points. I want to stress, however, that this notion of time points is purely formal and not intended to represent the occurrence of events. Events occur at t-locations, not at time points. Furthermore, the notion of t-location has an intrinsic conceptual and empirical indeterminacy. Nevertheless, one can think of time points as those points in time when new information about events, and consequently about time, becomes available.

the fact that each event has some intrinsic duration. I might be mentioned, however, that the idea that time is created by the occurrence of events has been discussed by philosophers, in particular, by A. N. Whitehead: „Realisierung ist das Werden von Zeit im Bereich der Ausdehnung. Ausdehnung ist der Komplex von Geschehnissen im Sinne ihrer Potentialitäten. In der Realisierung wird die Potentialität Wirklichkeit. Aber das potentielle Muster benötigt einen Zeitschnitt; und der Zeitschnitt muss als ein epochales Ganzes zutage treten, was durch die Realisierung des Musters erfolgt. Zeit ist also die Abfolge von Elementen, die an sich teilbar und benachbart sind. Ein Zeitschnitt, der zeitlich wird, führt dadurch die Realisierung im Hinblick auf ein dauerndes Objekt herbei. Verzeitlichung ist Realisierung. Verzeitlichung ist nicht ein weiterer kontinuierlicher Prozess. Sie ist eine atomistische Abfolge. Daher ist die Zeit atomistisch (d.h. epochal), auch wenn das, was verzeitlicht wird, teilbar ist.“ (Whitehead 1984, p. 152) For critical discussion, see Sipfle 1971.

[15] This formulation is used to mean an event occurring in an interval $]\,t_l, t_u\,]$ with $t_l < t < t_u$.

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